Decentralized-Market Design

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Financial markets

- Financial markets are
  - Imperfectly competitive
  - Fragmented/decentralized

- Parallel developments in other markets

  Production, labor, international trade, monetary policy, ...

  (Surveyed, e.g., in the special issue of Journal of Economic Perspectives in the Summer’ 2019, and the special issue of Journal of Monetary Economics in May 2021 re: the Carnegie-Rochester-NYU Conference)
Financial markets

- Financial markets are

  - **Imperfectly competitive**
    
    Dominated by institutional investors
    
    Investors rely on price impact estimation
    
    Price impact costs dominate explicit trading costs

  - **Fragmented/decentralized**
    
    Essentially all asset classes
    
    New online marketplaces:
    
    TradeWeb.com, BondDesk.com, MarketAxess.com, BrokerTec, eSpeed
This talk

Decentralized-market design

I. Has challenged the methods we have relied on

II. Has already significantly advanced
Models so far

- Imperfectly competitive markets
  
  At least since Wilson (1979), Glosten and Milgrom (1985), Kyle (1985, 1989), ...

- Decentralized/fragmented markets
  

New focus

Some recent questions in policy/regulation:

- Mandate to clear assets in a centralized platform rather than OTC
- Standardization
- Benchmark manipulation (LIBOR)
- Privacy
- Pre-trade and post-trade transparency
- Trading technology
- Alternative m.c. arrangements
- Weaknesses in design exposed during the pandemic
- Proposal to eliminate intermediaries

Focus has changed:
Initially: Inefficiencies due to market fragmentation

More recently: When is centralized trading efficient?
New focus

The literature has shown: If suitably designed, decentralized market can

- Be more efficient
- Improve distribution of risk
- Simplify the design for market participants
- Be more stable

Work ahead: Design principles for decentralized/fragmented markets

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Model: market

- Uniform-price double auction
  

Decentralized-Market Design

Model: market

- Uniform-price double auction
  

- \( K \) risky assets with payoffs \( \mathcal{N}(d, \Sigma) \)

- \( I \) traders
  
  \[
  \max_{q^i(p): \mathbb{R}^K \rightarrow \mathbb{R}^K} E[d \cdot (q^i + q_0^i) - \frac{\alpha^i}{2} (q^i + q_0^i) \cdot \Sigma (q^i + q_0^i) - p \cdot q^i | q_0^i]
  \]

  Trader \( i \) initially holds \( q_0^i = (q_{0,i})_k \in \mathbb{R}^K \) and trades \( q^i = (q_{i,k})_k \in \mathbb{R}^K \)

  \( \{q_{0,i,k}\}_{i,k} \) independent across \( i, k \)

- Market: A market is **centralized** if there is a single market clearing for all traders and assets and **decentralized** otherwise

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Decentralized markets

Consider the “centralized market” assumption

(1) Complete participation (w.r.t. traders and assets)
Decentralized markets

Consider the “centralized market” assumption

1. **Complete participation** (w.r.t. traders and assets)

2. **Complete conditioning** (of (net) demands)

   ▶ Contingent schedules:

   \[ q^i_k(p_1, \cdots, p_K) : \mathbb{R}^K \rightarrow \mathbb{R} \quad \forall k \in K \]

   ▶ Uncontingent schedules:

   \[ q^i_k(p_k) : \mathbb{R} \rightarrow \mathbb{R} \quad \forall k \in K \]

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\(^4\) Studied by Cespa (2004), Chen and Duffie (2021), Rostek and Yoon (2021), Wittwer (2021).
Equilibrium in a centralized market

- A demand profile \( \{q^i(p)\}_i \) is a (linear) BNE if and only if for each \( i \):

  (i) Optimization by trader \( i \):

  \[
  d - \alpha^i \sum (q^0_i + q^i) = p + \Lambda^i q^i \quad \forall p \in \mathbb{R}^K
  \]

  where \( \Lambda^i \equiv \frac{dp}{dq^i} = (\frac{dp_1}{dq^i_1}, \ldots, \frac{dp_K}{dq^i_K})_{k,l} = \begin{bmatrix}
  \frac{dp_1}{dq^i_1} & \cdots & \frac{dp_K}{dq^i_K} \\
  \vdots & \ddots & \vdots \\
  \frac{dp_1}{dq^i_K} & \cdots & \frac{dp_K}{dq^i_K}
\end{bmatrix} \); hence, trader \( i \) submits

  \[
  q^i(p, \Lambda^i) = (\alpha^i \sum + \Lambda^i)^{-1}(d - p - \alpha^i \sum q^0_i) \quad \forall p \in \mathbb{R}^K
  \]

  (ii) price impact of trader \( i \) is characterized by:

  \[
  \Lambda^i = -\left( \sum_{j \neq i} \frac{\partial q^i}{\partial p} \right)^{-1} = \left( \sum_{j \neq i} (\alpha^j \sum + \Lambda^j)^{-1} \right)^{-1}
  \]

- Solution: \( \Lambda^i = \beta^i \alpha^i \sum \)
Market that clears assets independently

- With uncontingent schedules, the market clears asset-by-asset:

\[ \sum_i q^i_k(p_k) = 0, \text{ determines equilibrium } p_k \text{ for each } k. \]

- Optimization problem of trader \( i \):

\[
\max_{\{q^i_k(p_k) : \mathbb{R} \to \mathbb{R}\}_k} E[d \cdot (q^i + q^i_0) - \frac{\alpha^i}{2} \left( q^i + q^i_0 \right) \cdot \Sigma(q^i + q^i_0) - p \cdot q^i | q^i_0],
\]

- F.O.C.: for all \( k \in K \) and \( p_k \):

\[
d_k - \alpha^i \sigma_{kk}(q^i_k + q^i_0,k) - \alpha^i \sum_{l \neq k} \sigma_{kl}(E[q^i_l | p_k, q^i_0] + q^i_0,l) = p_k + \lambda^i_k q^i_k \quad \forall p_k \in \mathbb{R}
\]

1. Demand for asset \( k \) depends on expected trades of assets \( l \neq k \) (rather than realized trades)
2. Price impact depends on inference (rather than fundamental risk \( \Sigma \) alone)
The fixed point in demands is equivalent to a fixed point in price impacts $\Lambda$!

$$q^i(p) = a^i - Bq_0^i - Cp,$$

where

$$a^i = C(d - (\alpha \Sigma - C^{-1}B)E[q_0]) + ((\alpha \Sigma + \Lambda)^{-1} \alpha \Sigma - B)(E[q_0] - E[q_0^i]),$$

intercept

$$B = ((1 - \sigma^2_0)(\alpha \Sigma + \Lambda) + (I - 1)\sigma^2_0 \Lambda')^{-1} \alpha \Sigma,$$

coefficient on $q_0^i$

$$C^{-1} = \left[(\alpha \Sigma + \Lambda)(BB')\right]_d \left[(BB')\right]^{-1}_d,$$

coefficient on $p$

$$\Lambda = \frac{1}{I - 1}(C^{-1})'.$$

price impact

$$a^i \equiv (a_k^i) \in \mathbb{R}^K, B \in \mathbb{R}^{K \times K}, \text{ and } C \in \mathbb{R}^{K \times K}$$

Market design analysis becomes a choice among traders’ price impact profiles
Why does independent market clearing matter?

▶ Innovation in decentralized markets
  • Synthetic products (e.g., ETFs, ETPs, derivatives)
  • Market-clearing technologies (e.g., by Etrade, Street Smart, Tradehawk)

▶ These instruments would be neutral (if well defined) if the market were centralized

▶ When assets do not clear jointly, spanning does not hold

▶ All innovations are redundant if and only if the asset payoffs are either perfectly correlated or independent
Welfare implications

Welfare

- With synthetic products, decentralized markets can be designed to be at least as efficient as the centralized market

- Joint market clearing is unnecessary and can be suboptimal

- Why?
New methods needed

Decentralized trading

(1) Weakens the role of spanning

• Structural methods (Hortacsu and McAdams (2010) and Kastl (2011), ...)
• Projected price impact

(2) Limits the scope for recursive analysis
New methods needed

Decentralized trading

(1) Weakens the role of spanning

(2) Limits the scope for recursive analysis

Typical approaches in analysis of dynamic markets

1. Dynamic trading with static inference
   • Markovian private information and symmetric traders (Vayanos (1999, 2001))
   • Prices are fully revealing in all rounds, or information is disclosed fully after each round (Vayanos (1999), Antill and Duffie (2017), Du and Zhu (2017a), Kyle, Obizhaeva, and Wang (2018), and Sannikov and Skrzypacz (2016))
   • Stationary equilibrium

2. Dynamic inference with static trading

3. Relax the assumptions about the state variables to keep track of e.g., oblivious equilibrium (Weintraub, Benkard, and Van Roy (2006)) and variants of self-confirming equilibrium (e.g., Fudenberg and Levine (1993, 2009), Dekel, Fudenberg, and Levine (1999), Cho and Sargent (2008), Battigalli, Cerreia Vioglio, Maccheroni, and Marinacci (2015), Pakes (2016))
New methods needed

- What is challenging with both dynamic (persistent) trading and inference?
  
  Dynamic trading alone $\rightarrow$ Backward recursion (Markovian)

  Dynamic inference alone $\rightarrow$ Forward recursion (Markovian)

  Dynamic trading and inference

  - Both backward and forward recursion
  - “Forecasting the forecasts of others” problem/ Curse of dimensionality

- Imperfect competition is key

- Equivalence with price impact: A nonrecursive approach
New methods needed

Decentralized trading

(1) Weakens the role of spanning

(2) Limits the scope for recursive analysis

(3) Calls for general matching models (theory of stability)

with substitutable and complementary agreements/contracts

(e.g., GSC, full substitutability)

and with externalities

(the formation of agreements between insurers and healthcare providers (e.g., Ho and Lee (2017)); between television networks and distributors (e.g., Crawford and Yurukoglu (2012)); between medical device manufacturers and hospitals (e.g., Grennan (2013)); ...)

Consider the **roommate problem** (Gale and Shapley, 1962)

- $I = \{1, 2, 3\}; X = \{x_{12}, x_{23}, x_{31}\}; X_1 = \{x_{12}, x_{31}\}, X_2 = \{x_{12}, x_{23}\}, X_3 = \{x_{23}, x_{31}\}$

- Each agent $i$ prefers rooming with $i + 1$ to $i - 1$, and prefers either to being unmatched

\[1: x_{12} \succ_1 x_{31}\]
\[2: x_{23} \succ_2 x_{12}\]
\[3: x_{31} \succ_3 x_{23}\]
E.g., suppose that when \( X \) is available,

- 1 believes 3 will choose \( x_{31} \), but 2 will not choose \( x_{12} \)
- 3 correctly believes that 1 will choose \( x_{31} \)
- 2 correctly believes neither 1 or 3 will choose an agreement with him
- (hence 1’s beliefs were correct)

\[
C_1(X_1|X_{-1}) = \{x_{31}\} \\
C_3(X_3|X_{-3}) = \{x_{31}\} \\
C_2(X_2|X_{-2}) = \emptyset
\]
New methods needed

Decentralized trading

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(4) Shifts the focus from efficiency to (re)distribution

(e.g., Dworczak, Kominers, Akbarpour (2020, 2021); surveys by Pathak (2016, Annual Reviews), Li (2017, Oxford REP), Kearns and Roth (2019))
Recent surveys

*IO Handbook*

Kastl (2020, *IJIO*)

Milgrom (2019, *Annual Reviews*)

Rostek and Yoon (2023, prepared for *JEL*)

Teytelboym, Li, Kominers, Dworczak, and Akbarpour (2021, *SJE*)

Weill (2020, *Annual Reviews*)
Thank You